Statistical Characterization of a Single-Photon Source Based on Stimulated FWM in Optical Fibers

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***Abstract*—We report on the theoretical statistical characterization of a single-photon source based on the stimulated four-wave mixing process inside an optical fiber. We quantify the photon statistics by means of the expectation number of photons generated on the signal and idler waves, $\langle \hat{N}_s \rangle$ and $\langle \hat{N}_i \rangle$ respectively, and Mandel's Q_s and Q_i parameters. The obtained solutions take into account the spontaneous and stimulated Raman contributions for the generation of the idler wave. Results show that the idler wave generated inside the fiber follows a super-Poissonian statistics. Findings also show that the signal wave change their own statistics from a Poissonian to a super-Poissonian, due to the interaction with the pump field and the optical fiber.**

***Index Terms*—Nonlinear optics, optical mixing, single-photon source.**

I. Introduction

Single-photons sources are essential for quantum communications, mainly for quantum key distribution technologies [1]. In optical fibers, the stimulated four-wave mixing process (FWM) can provide a way to generate single-photons already inside the optical fiber [2], [3]. Stimulated FWM in optical fibers is a third order nonlinear process, and this process occurs when two photons from an optical pump are annihilated and two new photons are created, one on the Stokes wave (known as signal) and another at the anti-Stokes (known as idler) [4]. Typically the stimulated FWM process is discussed independently of the Raman scattering process. However, both nonlinear process occurs inside the optical fiber [5]. In the Raman scattering process, frequencies from the pump are shifted to lower frequencies (Stokes lines) and to higher frequencies (anti-Stokes lines), through molecular vibrations [4], [6].

In this paper we analyze the combined processes of stimulated FWM and the Raman scattering effect. Moreover, we consider the phonon reservoir inside the fiber, which give rises to noise photons emitted by thermal excitation [7], [8]. We study the FWM process in the single-pump configuration. The photon statistics is studied for both signal and idler fields, through the calculation of the Mandel Q parameter [8]–[10]. That parameter measures the deviation of the photon number statistics from the Poisson distribution. A Mandel Q parameter higher than zero means that the variance of the photon number is higher than the average photon number, whereas a negative Q represents the situation in which the variance is smaller than the average [8]–[10]. We examine how the transmission through the fiber affects the photon statistics of the signal wave, as well the statistics of the idler wave generated inside the optical fiber.

This paper contains four sections. Section II reviews the quantum formalism of the evolution of the signal and idler annihilation operators inside the fiber. In section III we introduce the Mandel's Q parameter, and describe the signal and idler statistics based on the value of that factor. The main results presented in this paper are summarized in section IV.

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II. Theoretical model

The quantum evolution of the signal and idler annihilation operators inside the fiber is given by the input/output relation [7], [11]

$$\hat{A}(L,\omega_u) = \Big(\alpha_u(L)\hat{A}(0,\omega_u) + \beta_u(L)e^{-2i\theta_p}\hat{A}^\dagger(0,\omega_v) + \hat{F}(L,\omega_u)\Big)\Theta(L)\,, \quad (1)$$

where $u,v=s$ for the signal field and $u,v=i$ for the idler wave, with $u \neq v$, $\Theta(L)=\exp\{i(k_p+\gamma P_0)L\}$, θ_p is the phase of the optical pump coherent field, and [7]

$$\alpha_u(L) = \Big(\cosh(g(\Omega_{up})L) + \frac{i\kappa(\Omega_{up})}{2g(\Omega_{up})}\sinh((g(\Omega_{up})L)\Big)\Phi, \quad (2)$$

$$\beta_u(L) = i\frac{\gamma\eta(\Omega_{up})}{g(\Omega_{up})}A_p^2\sinh((g(\Omega_{up})L)\Phi\,, \quad (3)$$

$$\hat{F}(L,\omega_u) = i\int_0^L \hat{m}(z,\Omega_{up})\Big(A_p e^{-i\theta_p}\alpha_u(L-z) - A_p^* e^{i\theta_p}\beta_u(L-z)\Big)dz\,, \quad (4)$$

where $\Omega_{up}=\omega_u-\omega_p$, $\hat{m}(z,\Omega_{up})$ is the Hermitian phase noise operator, k_p is pump propagation constant, γ is fiber nonlinear parameter, and $P_0 = |A_p|^2$ is the input pump power. In (1)-(4)

$$g(\Omega_{up})^2 = (\gamma\eta(\Omega_{up})P_0)^2 - (\kappa(\Omega_{up})/2)^2 \,, \quad (5)$$

$$\kappa(\Omega_{up}) = \Delta\beta + 2\gamma P_0(\epsilon(\Omega_{up}) - 1) \,, \quad (6)$$

$$\Phi = \exp\{(-i(k_v - k_u)/2)L\} \,, \quad (7)$$

$$\epsilon(\Omega_{up}) = \eta(\Omega_{up}) + 1 = 2 - f_R + f_R\Big(\tilde{R}_a(\Omega_{up}) + \tilde{R}_b(\Omega_{up})\Big), \quad (8)$$

where $\Delta\beta$ is the phase-matching condition given by [4]

$$\Delta\beta = (\omega_p - \omega_0)(\omega_p - \omega_s)^2\beta_3 + \frac{1}{2}(\omega_p - \omega_s)^2 \times \Big[(\omega_p - \omega_0)^2 + \frac{}{1}6(\omega_p - \omega_s)^2\Big]\beta_4 \,, \quad (9)$$

where ω_0 is the frequency zero-dispersion of the fiber, β_3 and β_4 are the third and fourth order dispersion coefficients. In (8) $f_R = 0.245$ represents the fractional contribution of the delayed Raman response to the nonlinear refractive index, and $\tilde{R}_a(\Omega_{up})$ and $\tilde{R}_b(\Omega_{up})$ are the isotropic and anisotropic Raman response, respectively, and defined in [5], [7], [12], [13]. In (1) the input/output operators satisfies the commutation relation [7]

$$\Big[\hat{A}(z,\omega_u), \hat{A}^\dagger(z,\omega_v)\Big] = 2\pi\delta(\omega_u - \omega_v) \,. \quad (10)$$

The noise operators satisfies the commutation relation [7], [8]

$$\Big[\hat{F}(z,\omega_u), \hat{F}^\dagger(z,\omega_v)\Big] = = 2\pi\, \mathrm{sgn}(\Omega_{up})(1 + |\beta_u(L)|^2 - |\alpha_u(L)|^2)\delta(\omega_u - \omega_v) \,, \quad (11)$$

and

$$\big[\hat{m}(z,\Omega_{up}), \hat{m}^\dagger(z,\Omega_{vp})\big] = 2\pi g_R(\Omega_{up})\delta(\Omega_{up} - \Omega_{vp}) \,, \quad (12)$$

where $g_R(\Omega_{up})$ is the Raman gain for linearly copolarized pumps [12].

In practice, the signal and idler optical fields are filtered spectrally using an optical filter, due to the fact that the photon counters are sensitive to a bundle of frequencies. In that case, the signal and idler optical fields are given by [7], [8]

$$\hat{A}_u(z,t) = \frac{1}{2\pi}\int d\omega_u H(\omega_u)\hat{A}(L,\omega_u)e^{-i\omega_u t} \,, \quad (13)$$

where $H(\omega_u)$ is the filter function centered at $\bar{\omega}_u$.

III. Photon Statistics

In this section, we analyze the photon statistics of the FWM process. We examine how the transmission through the fiber affects the photon statistics of the signal wave. We also study the photon number distribution of the idler wave generated inside the optical fiber. The statistical properties of the fields are characterized by Mandel's Q parameter [8], [9]

$$Q_u = \frac{\langle : \hat{N}_u^2 : \rangle - \langle \hat{N}_u \rangle^2}{\langle \hat{N}_u \rangle} \,, \quad (14)$$

where $\langle \hat{N}_u \rangle$ is the expectation value for the dimensionless number operator, and $\langle :\hat{N}_u^2: \rangle$ is the normally ordered second moment [8]. A $Q_u = 0$ corresponds to coherent light, while $Q_u > 0$ or $Q_u < 0$ determines super-Poissonian or sub-Poissonian statistics, respectively [14]. Thermal light and chaotic light, are two examples of super-Poissonian statistics, while sub-Poissonian statistics has no classical counterpart, it is regarded as a quantum feature of the optical field [14]. Single-photons from a highly attenuated laser optical field are commonly used in quantum key distribution systems in optical fibers, and that kind source presents a null Mandel Q factor, since follows a Poissonian statistics [1]

The expectation values for the Mandel parameter are [8]

$$\langle \hat{N}_u \rangle = \frac{1}{(2\pi)^2}\int_{t_0}^{t_0+T_0} dt \int d\omega_u \int d\omega_u^{'} H^*(\omega_u)H(\omega_u^{'}) \times \langle \hat{A}^\dagger(L,\omega_u)\hat{A}(L,\omega_u^{'})\rangle e^{i(\omega_u - \omega_u^{'})t} \,, \quad (15)$$

and

$$\langle : \hat{N}_u^2 : \rangle = \frac{1}{(2\pi)^4}\int_{t_0}^{t_0+T_0} dt \int_{t_0}^{t_0+T_0} dt^{'} \int d\omega_u \int d\omega_u^{'} \int d\omega_u^{''} \times \int d\omega_u^{'''} e^{i(\omega_u - \omega_u^{''})t} e^{i(\omega_u^{'} - \omega_u^{'''})t} \times H^*(\omega_u)H^*(\omega_u^{'})H(\omega_u^{''})H(\omega_u^{'''}) \times \langle \hat{A}^\dagger(L,\omega_u)\hat{A}^\dagger(L,\omega_u^{'})\hat{A}(L,\omega_u^{''})\hat{A}(L,\omega_u^{'''})\rangle \,. \quad (16)$$

In (15) and (16) T_0 is the detector integration time, and the expectation values inside the integrals are over a product state that comprises the incident states to the fiber, as well as states $|f\rangle$ that accounts the phonon reservoir inside the fiber [7], [8]. We take at the input of the fiber the signal field as a continuous mode coherent state and the idler wave as a vacuum state [15]–[17]

$$|\Psi\rangle = |f\rangle|0_i\rangle \exp\{\sigma_s\hat{A}^\dagger(0,\omega_s) - \sigma_s^*\hat{A}(0,\omega_s)\}|0_s\rangle \,, \quad (17)$$

where $\sigma_s = \sqrt{2\pi\bar{n}_{s0}}\exp\{-i\theta_s\}$, and $\bar{n}_{s0}$ is the mean photon flux per unit of angular frequency ω_s, given by $\bar{n}_{s0} = 2\pi P_s/\hbar\omega_s^2$, and P_s is the input signal power [15]–[17].
The expectation value in (1) for the noise operator is [7], [8]

$$\langle\Psi|\hat{F}^\dagger(L,\Omega_{up})\hat{F}(L,\Omega_{vp})|\Psi\rangle = 2\pi\left(n_{th}(\Omega_{up}) + \Xi(-\Omega_{up})\right) \times \mathrm{sgn}(\Omega_{up})(1 + |\beta_u(L)|^2 - |\alpha_u(L)|^2)\delta(\Omega_{up} - \omega_{vp}) \,, \quad (18)$$

where Ξ is the Heaviside function.

The multiple integrals in (15) and (16) can be evaluate taking the advantage that the filter bandwidth is so small that the expectation values does not appreciable change across the effective passband of the filters [8], [9]. In that case, (15) and (16) can be solved and the solutions for the signal and idler fields are

$$\langle \hat{N}_u \rangle \cong (\Delta\nu_u T_0)\Big[\left(\Xi(-\bar{\Omega}_{up}) + \bar{n}_{\bar{so}}\right)|\bar{\beta}_u|^2 + \mathrm{sgn}(\bar{\Omega}_{up}) \times \left(n_{th}(\bar{\Omega}_{up}) + \Xi(-\bar{\Omega}_{up})\right)(1 + |\bar{\beta}_u(L)|^2 - |\bar{\alpha}_u(L)|^2)\Big], \quad (19)$$

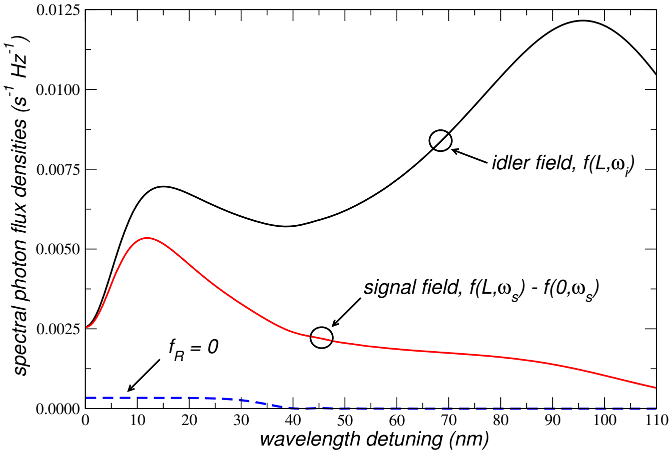

Fig. 1. Photon flux spectral density given by (15), $\langle \hat{N}_u \rangle / \Delta\nu_u T_0$, as function of the wavelength separation between pump and signal fields. The parameters used are: $\beta_3 = 0.1$ ps^3/km, $\beta_4 = 10^{-4}$ ps^4/km, $\gamma = 2$ W^{-1}/km, $P_0 = 15$ mW, $P_s = 1$ μW, $\lambda_p = \lambda_0 = 1550.918$ nm, $T = 300$ °K, and $L = 600$ m.

where $\Delta\nu_u$ is the optical filter filter bandwidth, $n_{th}(\Omega) = [e^{\hbar\Omega/k_B T} - 1]^{-1}$, and

$$\langle : \hat{N}_i^2 : \rangle \cong (\Delta\nu_i T_0)^2 \Big[(2 + 4\bar{n}_{\bar{s}o} + \bar{n}_{\bar{s}o}^2)|\bar{\beta}_i|^4 + (1 + \bar{n}_{\bar{s}o})|\bar{\beta}_i|^2 \times \left(n_{th}(\bar{\Omega}_{ip}) + 1\right) \left(|\bar{\alpha}_i(L)|^2 - |\bar{\beta}_i(L)|^2 - 1\right) + 2(n_{th}(\bar{\Omega}_{ip}) + 1)^2 (|\bar{\alpha}_i(L)|^2 - |\bar{\beta}_i(L)|^2 - 1)^2 \Big], \quad (20)$$

$$\langle : \hat{N}_s^2 : \rangle \cong (\Delta\nu_s T_0)^2 \Big[\bar{n}_{\bar{s}o} |\bar{\alpha}_s(L)|^4 + 4\bar{n}_{\bar{s}o} |\bar{\alpha}_s(L)|^2 |\bar{\beta}_s(L)|^2 + 2|\bar{\beta}_s(L)|^4 + 4\bar{n}_{\bar{s}o} |\bar{\alpha}_s(L)|^2 n_{th}(\bar{\Omega}_{sp})(1 + |\bar{\beta}_s(L)|^2 - |\bar{\alpha}_s(L)|^2) + 4|\bar{\beta}_s(L)|^2 n_{th}(\bar{\Omega}_{sp})(1 + |\bar{\beta}_s(L)|^2 - |\bar{\alpha}_s(L)|^2) + 2n_{th}(\bar{\Omega}_{sp})^2 (1 + |\bar{\beta}_s(L)|^2 - |\bar{\alpha}_s(L)|^2)^2 \Big]. \quad (21)$$

In (19)-(21) the functions $\bar{\alpha}_u(L)$ and $\bar{\beta}_u(L)$ are given by (2) and (3), respectively, with ω_u replaced by the filter centered frequency $\bar{\omega}_u$.

In Fig. 1 we plot the photon flux spectral density given by (15), $\langle \hat{N}_u \rangle / \Delta\nu_u T_0$, as function of the wavelength separation between pump and signal fields, for two cases $f_r = 0$ (without Raman scattering), and $f_R = 0.245$ (considering the Raman scattering). It can be seen that, the Raman scattering is the dominant process in the generation of the photons inside the fiber. The flux of photons generated inside the fiber due only the FWM process is much smaller than the noise photon flux due to the spontaneous and stimulated Raman scattering.

In Fig. 2 we plot the evolution of the Mandel Q_u parameter, given by (14), as a function of the wavelength separation between pump and signal waves, in units of $\Delta\nu_u T_0$. Results show that both signal and idler waves present a $Q_u > 0$, which means that the fields are described by a super-Poissonian statistics. From plot (a) we can see that the major contribution for Q_i arises thermal part of (19) and (20). When we consider only the FWM process, $f_R = 0$, the Mandel Q_i parameter

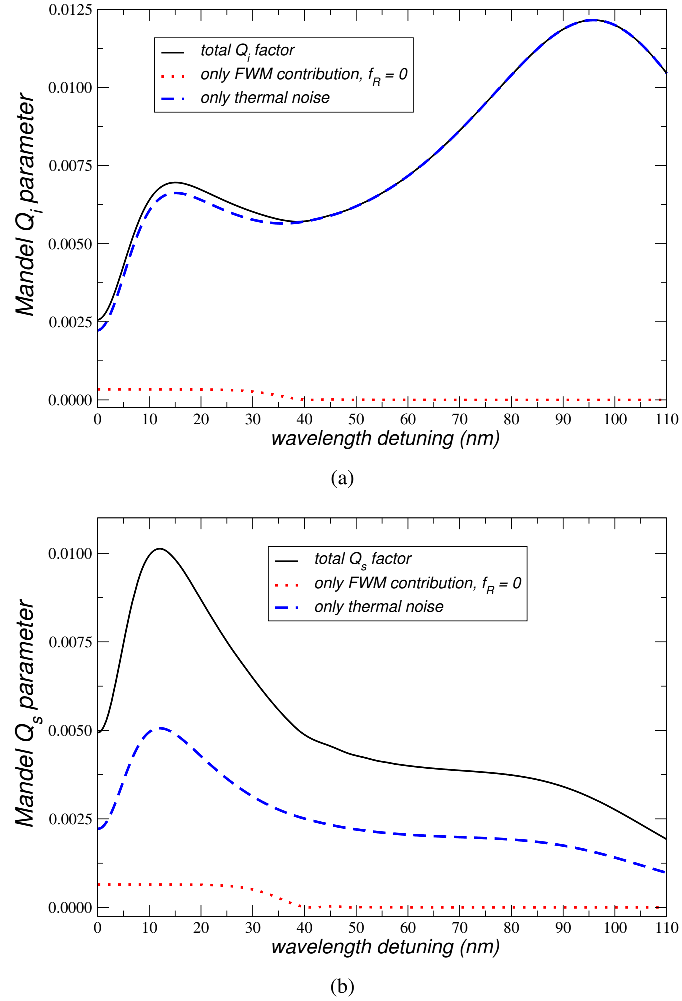

Fig. 2. Mandel Q_u parameter, given by (14), as a function of $\lambda_p - \lambda_s$, in units of $\Delta\nu_u T_0$. Plot (a) represents the idler wave, whereas plot (b) shows the evolution of the signal field. Parameters are the same as in Fig. 1.

is almost null. It can be seen in plot (b) that, the signal field changes their own statistics, since at the fiber input that field is in a coherent state, given by (17), and at the output of the optical fiber we obtain $Q_s > 0$, which represents a super-Poissonian statistics.

Figure 3 shows the Mandel Q_i parameter as a function of pump-signal wavelength detuning for three different temperatures. It can be seen in Fig. 3 that with increasing the temperature the Mandel Q_i parameter also increases, mainly for small wavelength detunings, less than 30 nm. For higher wavelength detuning, results in Fig. 3 show that the main contribution for the Mandel parameter arises from the Raman gain curve, that is present in first part of (19) and (20). Results also show that, when temperature increase the thermal noise contribution to the idler statistics also increase.

Single-photons sources with $Q > 0$ can be used in quantum communications experiments, for instance entangled photon pairs generated by spontaneous FWM present a thermal statistics (for a single mode of the field) are commonly used in quantum key distribution systems [1], [7].

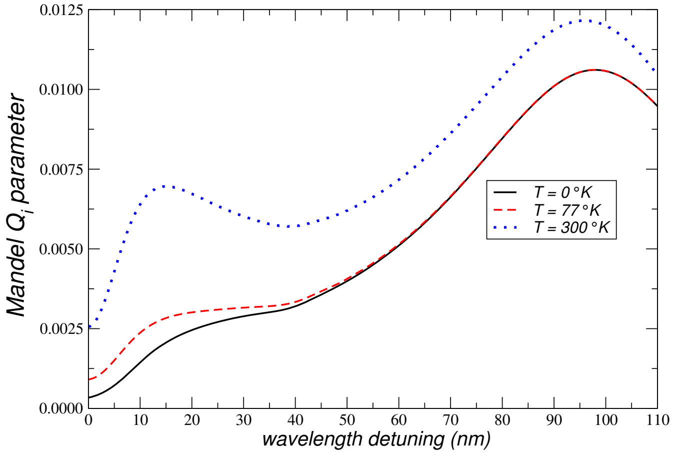

Fig. 3. Mandel Q_i parameter as a function of $\lambda_p - \lambda_s$, for three different temperatures, in units of $\Delta\nu_u T_0$. Other parameters are the same as in Fig. 1.

IV. Conclusion

In summary, we investigate the statistics of the FWM process in optical fibers. We analyze the statistics in terms of the Mandel parameter, for both signal and idler waves. We show that the stimulated and spontaneous Raman scattering dramatically change the evolution of the Q_u parameter with the wavelength detuning, when compared with the case $f_R = 0$. We verify that the idler wave generated inside the fiber obey to a super-Poissonian statistics. We also show theoretically that the signal field change their own statistics from a Poissonian at $L = 0$ to a super-Poissonian at the end of the fiber.

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