Nonlinear Phase and Parametric Gain in Optical Fiber Microwires

Gil G. Fernandes, Álvaro J. Almeida, and Armando N. Pinto

***Abstract*—We study the evolution of the nonlinear phase in a optical fiber microwire (OFM), and the parametric gain obtained through the stimulated four-wave mixing (SFWM) process. A theoretical analysis is performed, and then the results are presented. The efficiency of the SFWM process depends on several parameters, such as the phase mismatch, Δk_0, and the nonlinear coefficient, γ. The zero-dispersion wavelength, λ_0, and the slope dispersion, S, domain the phase mismatch, as the powers, P_0, are extremely important for the parametric gain, g, and the nonlinear phase. We perform a detailed analysis on these quantities. The efficiency of the process will be as maximum for $\Delta k_0 = \gamma P_0$.**

***Index Terms*—Optical Mixing, Nonlinear Phase, Parametric Gain, Phase Mismatch, Optical Microwires.**

I. INTRODUCTION

THE four-wave mixing (FWM) process has been exhaustively studied in the last few years, being several applications for this process already proposed. The FWM process can be used to implement fiber-optic parametric amplifiers [1]–[3], optical regenerators [4], wavelength converters [5]–[7], and single and entangled photon-pair sources [8]–[13]. However, the low nonlinear parameter in standard optical fibers is a limitation for possible applications of this process. Recently, several nonlinear fibers have been developed with the nonlinear parameter hundreds of times larger [4], [14]. In order to obtain the maximum efficiency of the FWM process, several conditions must be verified. Firstly, the phase mismatch must be reached. In this way the pump must be launched at the zero-dispersion wavelength of the fiber. However, it could not be enough for maximizing the FWM efficiency. The nonlinearities present in the fiber can deteriorate the signal during the propagation, changing the polarization, the shape of the pulse, and can induce an additional nonlinear phase in the pulse [3], [15], [16]. Then, the nonlinear phase induced for the nonlinear effects has to be taken into account, and the maximum gain can occur when the phase mismatch is different from zero. In this way, the nonlinear interaction can have a central role [3].

The influence of the nonlinear phase in the FWM process can be analyzed in optical fiber microwires (OFMs), a particular kind of nonlinear fibers. OFMs have a special interest due to the high nonlinear coefficients [17]. These fibers have been used recently for optical parametric amplification [4]. The nonlinear phase acquired by the wave can be important for the parametric energy transfer between the pump and the signal. This parameter can be very important in optical parametric amplification and regeneration devices, specially when high nonlinear fibers are used [3], [4].

In this paper, the variation of the parametric gain in OFMs is analyzed as a function of the phase induced by the nonlinear effects. Subsequently, the evolution of the nonlinear phase of the waves that can input in a OFM is analyzed. We also study how the nonlinear phase can be maximized, in order to obtain a maximum gain.

This paper contains four sections. In Section II, we describe the theoretical formulation for the nonlinear phase and the parametric gain in OFMs. In Section III, we present the results obtained for an OFM. Finally, in Section IV we show the main conclusions of this work.

II. THEORETICAL DESCRIPTION

The stimulated four-wave mixing (SFWM) process can be achieved by sending a pump and a signal waves into an optical fiber, where the signal will be amplified and the idler wave generated. During the propagation in the fiber, the pump, that has the highest power, will transfer energy to the signal and the idler, which induces a nonlinear phase, that is mainly due to cross-phase modulation (XPM) and self-phase modulation (SPM) [3].

The pump, signal, idler and the phase evolutions in a optical fiber, can be described as [18],

$$\frac{\mathrm{d}P_{(\mathrm{p,s,i})}}{\mathrm{d}z} = f(P_\mathrm{p}, P_\mathrm{s}, P_\mathrm{i}, \theta)\,, \text{ and } \quad \frac{\mathrm{d}\theta}{\mathrm{d}z} = f(P_\mathrm{p}, P_\mathrm{s}, P_\mathrm{i}, \theta)\,, \quad (1)$$

which can be used to describe the transfer of power between the pump, the signal and the idler, and how the nonlinear phase of the wave varies. The total nonlinear phase takes into account several terms, as described in the following equation [18],

$$\theta = \Delta k_0 z + \phi_\mathrm{s} + \phi_\mathrm{i} - 2\phi_\mathrm{p}\,, \quad (2)$$

where ϕ_p, ϕ_s and ϕ_i are the phases of the pump, the signal and the idler, respectively, Δk_0 is the phase mismatch, and z is the fiber length.

An efficient SFWM process will lead to a substantial parametric gain, g, that depends on the pump and signal

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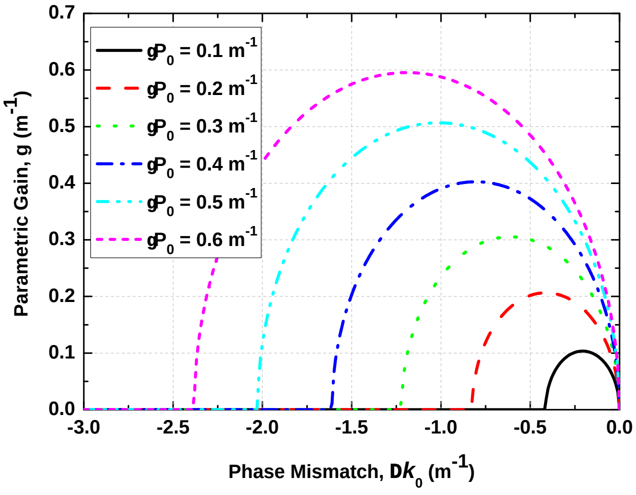

Fig. 1. Parametric gain, g, as a function of the phase mismatch, Δk_0, for several values of γP_0.

powers, the nonlinear parameter, and the phase mismatch. The parametric gain can be defined as [3],

$$g = \sqrt{(\gamma P_0 r)^2 - (\kappa/2)^2}\,, \tag{3}$$

where κ is the effective phase mismatch, that is given by,

$$\kappa = \Delta k_0 + \gamma(P_\text{p} + P_\text{s})\,, \tag{4}$$

and

$$r = 2\sqrt{P_\text{p} P_\text{s}}/P_0\,, \qquad \text{with} \qquad P_0 = P_\text{p} + P_\text{s}\,. \tag{5}$$

Thus, the maximum gain is obtained for the phase mismatch between the pump and the signal, that is equal to γP_0. As the OFMs are similar structures as standard optical fibers, we will use this theory to describe their behavior, that, unless some specific conditions, still remains valid [19].

III. NONLINEAR PHASE AND PARAMETRIC GAIN IN OFMS

The SPM and the XPM can induce an additional phase in the waves that interact in the SFWM process. The parametric gain changes with γ, P_p and P_s, and Δk_0, according to (3). In Fig. 1, we show the variation of the parametric gain, g, with the phase mismatch, Δk_0, for several values of γP_0. The maximum gain, $g_\text{max} = \gamma P_0$, occurs at $\kappa = 0$, or also at $\Delta k_0 = -2\gamma P_0$. From this figure, it can be seen that the peak gain is shifted from $\Delta k_0 = 0$, and that shift increases with g_max. This is due to the contribution of XPM and SPM to the phase mismatch, that can be much important in OFMs, in which γ changes strongly with the radius, and consequently the phase acquired by the signals also changes.

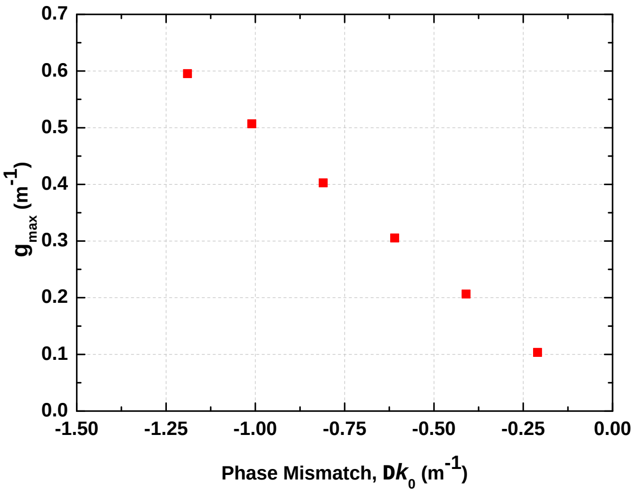

Fig. 2. Maximum gain, g_max, as a function of the phase mismatch, Δk_0.

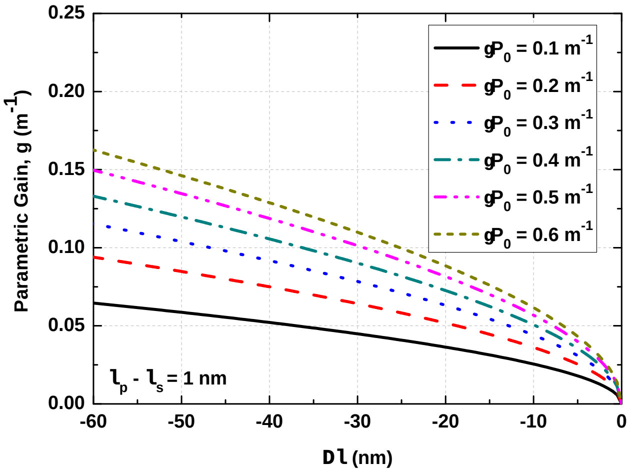

Fig. 3. Parametric gain, g, as a function of the wavelength separation, $\Delta\lambda$, for several values of γP_0, when the wavelength separation between the pump and signal waves is equal to 1 nm.

In Fig. 2 it can be seen the variation of the maximum gain, g_max, with the phase mismatch, Δk_0. A linear behavior is verified between the two quantities, as expected. The nonlinear phase induced in the signal decrease with the γP_0. In this way for moderated pump powers the nonlinear phase induced by the SPM and the XPM can be neglected and the phase mismatch, Δk_0, can be written in the same way as it is around the zero-dispersion wavelength, and is expressed

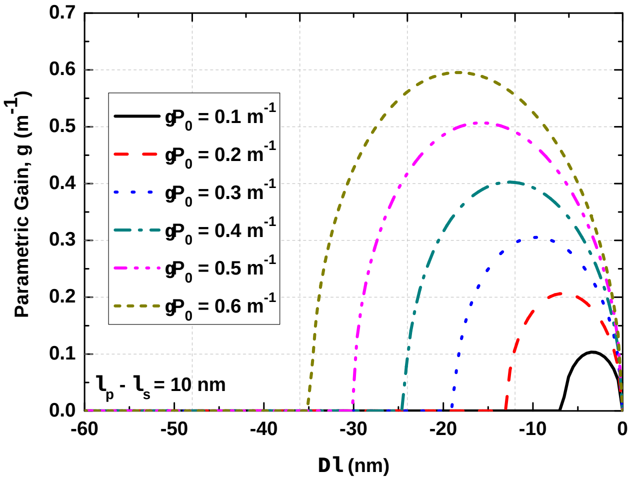

Fig. 4. Parametric gain, g, as a function of the wavelength separation, $\Delta\lambda$, for several values of γP_0, when the wavelength separation between the pump and signal waves is equal to 10 nm.

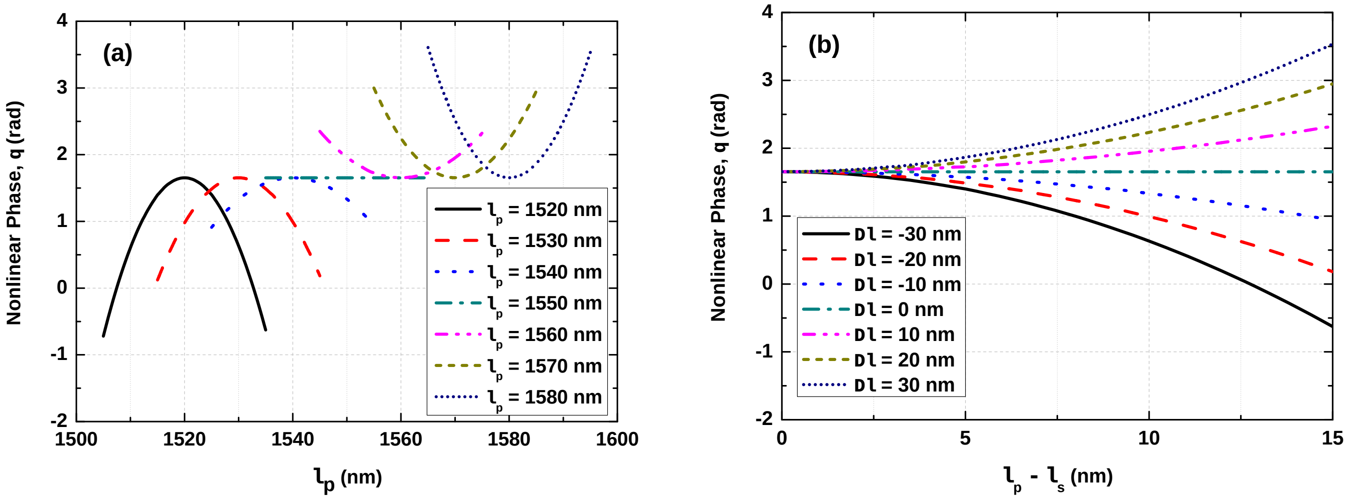

Fig. 5. (a) Nonlinear phase variation with the pump wavelength, when the pump is sent at different separations from the λ_0. (b) Nonlinear phase variation with the separation between the pump and the signal, $\lambda_\mathrm{p} - \lambda_\mathrm{s}$.

as [18], [19],

$$\Delta k_0 = -\frac{2\pi c}{\lambda^2}\frac{\mathrm{d}D}{\mathrm{d}\lambda}(\lambda_\mathrm{p} - \lambda_0)(\lambda_\mathrm{p} - \lambda_\mathrm{s})^2\,, \tag{6}$$

where λ_p and λ_s are the pump and signal wavelengths, respectively, λ_0 is the zero-dispersion wavelength of the fiber, and D is the dispersion.

In Fig. 3, we plot the parametric gain, g, as a function of $\Delta\lambda$, when the signal is launched 1 nm next to the pump, for several values of γP_0. In Fig. 4, we show the results for the case when the signal is launched 10 nm next to the pump, for the same γP_0 values, verifying considerably differences in relation to the previous case. As can be seen from Figs. 3 and 4, if the signal was sent farther to the pump, g_max is found closer to $\Delta k_0 = 0$, and then, a more efficient FWM process can be obtained.

From equation (3), we can see that the ideal phase mismatch, in order to maximize the gain, changes strongly in a OFM with the separation between the pump and the signal. The variation of the phase mismatch, due to the variation of dispersion and the slope dispersion, is reflected on the other hand, in the variation of the parametric gain.

In order to obtain the variation of the nonlinear phase after propagation through a single-mode OFM, we have solved numerically the set of equations in (1). From that equations, it can be obtained the nonlinear phase at the output of the OFM, assuming an initial phase, $\theta = \pi/2$ [18]. In order to work around the zero-dispersion wavelength used in the telecommunications band, the OFMs must have a diameter, $\Phi \approx 1.2$ μm. In this way, the parameters considered for the OFM are the following: length, $L = 1$ m, nonlinear parameter, $\gamma = 82$ W^{-1}km^{-1}, zero-dispersion wavelength, $\lambda_0 = 1550$ nm, and slope dispersion at λ_0, $S(\lambda_0) = -0.79$ ps/nm^2-km. In Fig. 5, we plot the nonlinear phase variation as a function of the pump wavelength, λ_p, and the wavelength separation, λ_p–λ_s. As can be seen from Fig. 5(a), the nonlinear phase has a parabolic behavior, with an increasing curvature radius, until it reaches the zero-dispersion wavelength, where it is considered infinity. This is because only the third-order terms in the expansion of the Taylor series are taken into account in (6). Indeed, the flat gain observed would be adequately corrected to more terms considered in the expansion. From Fig. 5(b), we can see in more detail how the parabolic curvature evolves with the wavelength separation between the pump and the signal, where $\Delta\lambda = \lambda_\mathrm{p} - \lambda_0$. The variation of the curvature of the phase can be used to find the zero-dispersion wavelength of an OFM. In this way, the zero-dispersion wavelength can be used to estimate the OFM diameter, and consequently the nonlinear parameter can be calculated [19].

IV. CONCLUSIONS

The variation of the nonlinear phase with the wavelength in a OFM was presented. This parameter can be used to determine the zero-dispersion wavelength of the OFM, in case of it is unknown, the efficiency of the SFWM process, and also its diameter. We have calculated the parametric gain in a OFM as a function of the phase mismatch and the separation between the wavelength of the pump and the signal, for several values of γP_0. An increase on this parameter also leads to an increase on the parametric gain, significantly. It was also verified that increasing the separation between the pump and the signal can have several advantages, mainly when one wants to determine the efficiency of the SFWM process. In the SFWM process, it was verified that launching the pump at the zero-dispersion wavelength is not the only condition to achieve maximum efficiency, but the additional nonlinear phase induced for the SPM and the XPM effects should be taken into account. This effect is specially important in OFMs, due to the high nonlinear coefficients. Thus, in order to maximize the efficiency of the SFWM process is necessary to ensure that $\Delta k_0 = \gamma P_0$.

REFERENCES

[1] Q. Lin, F. Yaman, and G. P. Agrawal, "Raman-induced polarization-dependent gain in parametric amplifiers pumped with orthogonally polarized lasers," *IEEE Photonics Technology Letters*, vol. 18, pp. 397–399, Jan. 2006.

[2] F. Yaman, Q. Lin, S. Radic, and G. P. Agrawal, "Fiber-Optic Parametric Amplifiers in the Presence of Polarization-Mode Dispersion and Polarization-Dependent Loss," *Journal of Lightwave Technology*, vol. 24, pp. 3088–3096, Aug. 2006.

[3] G. Agrawal, *Nonlinear Fiber Optics, Fourth Edition (Optics and Photonics)*, 4th ed. Academic Press, Oct. 2006.

[4] M. D. Pelusi, F. Luan, E. Magi, M. R. Lamont, D. J. Moss, B. J. Eggleton, J. S. Sanghera, L. B. Shaw, and I. D. Aggarwal, "High bit rate all-optical signal processing in a fiber photonic wire," *Optics Express*, vol. 16, pp. 11 506–11 512, Jul. 2008.

[5] J. Yu and P. Jeppesen, "Wavelength conversion by use of four-wave mixing in a novel optical loop configuration," *Opt. Lett.*, vol. 25, no. 6, pp. 393–395, Mar 2000.

[6] K. Schulze, M. Petersen, J. Herrera, F. Ramos, and J. Marti, "Optimisation of 40 gb/s wavelength converters based on four-wave mixing in a semiconductor optical amplifier," *Optics Communications*, vol. 276, no. 1, pp. 158–160, 2007.

[7] F. Morichetti, A. Canciamilla, C. Ferrari, A. Samarelli, M. Sorel, and A. Melloni, "Travelling-wave resonant four-wave mixing breaks the limits of cavity-enhanced all-optical wavelength conversion," *Nature Communications*, vol. 2, May 2011.

[8] X. Li, P. L. Voss, J. E. Sharping, and P. Kumar, "Optical-Fiber Source of Polarization-Entangled Photons in the 1550 nm Telecom Band," *Physical Review Letters*, vol. 94, no. 5, pp. 053 601:1–053 601:4, Feb. 2005.

[9] N. A. Silva, N. J. Muga, and A. N. Pinto, "Influence of the Stimulated Raman Scattering on the Four-Wave Mixing Process in Birefringent Fibers," *IEEE/OSA Journal of Lightwave Technology*, vol. 27, no. 22, pp. 4979–4988, Nov. 15 2009.

[10] ——, "Effective Nonlinear Parameter Measurement Using FWM in Optical Fibers in a Low Power Regime," *IEEE Journal of Quantum Electronics*, vol. 46, no. 3, pp. 285–291, Mar. 2010.

[11] A. J. Almeida, N. A. Silva, N. J. Muga, and A. N. Pinto, "Fiber-optical communication system using polarization-encoding photons," *15th European Conference on Networks and Optical Communications*, Jun. 2010.

[12] A. J. Almeida, S. R. Carneiro, N. A. Silva, N. J. Muga, and A. N. Pinto, "Polarization-entangled photon pairs using spontaneous four-wave mixing in a fiber loop," in *Proceedings of the 2011 International Conference on Computer as a Tool and 8th Conference on Telecommunications*, Apr. 2011.

[13] A. J. Almeida, N. A. Silva, N. J. Muga, and A. N. Pinto, "Single-photon source using stimulated fwm in optical fibers for quantum communication," in *Proceedings of 1st International Conference on Applications of Optics and Photonics*, May 2011.

[14] E. C. Mägi, L. B. Fu, H. C. Nguyen, M. R. Lamont, D. I. Yeom, and B. J. Eggleton, "Enhanced Kerr nonlinearity in sub-wavelength diameter As_2Se_3 chalcogenide fiber tapers," *Optics Express*, vol. 15, pp. 10 324–10 329, 2007.

[15] J. Breguet, A. Muller, and N. Gisin, "Quantum Cryptography with Polarized Photons in Optical Fibres: Experiment and Practical Limits," *Journal of Modern Optics*, vol. 41, pp. 2405–2412, Dec. 1994.

[16] W. T. Buttler and S. K. Lamoreaux, "Quantum cryptography with polarization, phase and time encoding," *ArXiv e-prints*, Aug. 2008.

[17] G. Brambilla, "Optical fibre nanowires and microwires: a review," *Journal of Optics*, vol. 12, no. 4, pp. 043 001:1–043 001:19, Apr. 2010.

[18] K. Inoue and T. Mukai, "Signal wavelength dependence of gain saturation in a fiber optical parametric amplifier," *Optics Letters*, vol. 26, pp. 10–12, Jan. 2001.

[19] G. G. Fernandes, A. J. Almeida, and A. N. Pinto, "Measurement of the diameter of an optical fiber microwire using stimulated four-wave mixing," in *Proceedings of the International Conference on Applications of Optics and Photonics*, May 2011.