

# Measurement of the Diameter of an Optical Fiber Microwire using Stimulated Four-Wave Mixing

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## ABSTRACT

We present a nondestructive technique for measuring the diameter of optical fiber microwires (OFMs), using the stimulated four-wave mixing (SFWM) process. The diameter of the OFMs is also estimated with high accuracy from its zero dispersion wavelength. A numerical analysis is performed, and the ideal parameters for easy experimental measurements are presented and discussed.

**Keywords:** Microwires, Nonlinear Optics, Zero-Dispersion Wavelength, Slope Dispersion, Nonlinear Coefficient.

## 1. INTRODUCTION

Optical fiber microwires (OFMs), presents itself a great interest for a wide range of applications, namely supercontinuum generation,<sup>1,2</sup> parametric amplification,<sup>3</sup> pulse compression,<sup>4</sup> sensors,<sup>5</sup> particle manipulation<sup>2</sup> and high Q-resonators.<sup>2</sup> For all the applications mentioned is important to know parameters like the diameter, the dispersion and the nonlinear coefficient. However, the small diameter, as well as the high difference between cladding and air index, lead to a strong confinement of the optical pulse. This means that the waveguide dispersion becomes dominant and the nonlinear coefficient increases. The OFMs characteristics as the dispersion and nonlinear coefficient, are difficult to determinate, first, because they are extremely dependent on the diameter, and second, because of the small length of these optical structures.<sup>1,2</sup>

In OFMs, and contrary to optical fibers, it is much difficult to measure dispersion and the nonlinear coefficient. The small length and a strong variation of the dispersion as a function of the diameter, are the two major limitations in the characterization of OFMs. Then, its understandable that the measurement techniques are considerable less than for optical fibers. However, was also previously reported the zero dispersion wavelength (ZDW) measurement using broadband-light interferometry.<sup>6</sup> This is a simple and reliable technique to measure the dispersion of a tapered fiber. However, due to light sources limitations, there is a lack of experimental data in the measurement region. The nonlinear pulse propagation is used to estimate the nonlinear coefficient, where the input and the output powers are monitored, and the spectral broadening due to self-phase modulation (SPM) is clearly observed.<sup>7</sup> However, this technique needs an OFM produced from high nonlinear refractive index and considerable input powers.<sup>7</sup> Supercontinuum generation is extremely sensible to these parameters and the efficiency of this process can be used to infer the ZDW and the nonlinear coefficient. Nevertheless, this technique need a femtosecond ultrafast laser.

In order to contribute for this area, we present a nondestructive technique to find the ZDW in OFMs that uses the stimulated four-wave mixing (SFWM) process. The high nonlinearities, as well as the known of the

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ZDW, lead to a rather more efficient FWM process. This process allows to measure with high accuracy and without the characteristic uncertainty of other direct methods, the ZDW, slope dispersion and the nonlinear coefficient. Once the ZDW is known, the OFM diameter can be calculated with high accuracy.

This paper is organized in four sections. In section 2, we present the theoretical considerations that need to be taken to describe this characterization technique. In section 3 we present an experimental setup to find the zero-dispersion wavelength, and consequently measure the OFM diameter. Finally, in section 4 we present our conclusions.

## 2. MODEL CONSIDERATIONS

The OFMs can be produced from standard optical fibers from a tapering process. In Fig. 2, the optical structure resultant is represented. It can be seen that this has three different regions: two tapered regions, where the radius changes; and one waist region or OFM, where the radius remains constant. Along the tapered regions the refractive index profile changes and the fundamental mode can be coupled for high order modes or even leaking modes. However, adiabatic tapered regions were considered to minimize the modal coupling and consequently the optical losses in these structures. Contrary to optical fibers, in OFMs the optical pulse is guided due to the high contrast of refractive indices between the cladding and the air. This leads to the strong guiding of the optical pulse.<sup>8</sup>

Along the tapered regions, the distribution of the optical field changes, and consequently, the nonlinear coefficient and the dispersion varies. In the OFM, the diameter remains constant, the high nonlinear coefficient and the null dispersion values improve and lead to an increase in the efficiency of the FWM process. In this way, the contribution of the tapered regions can be neglected.

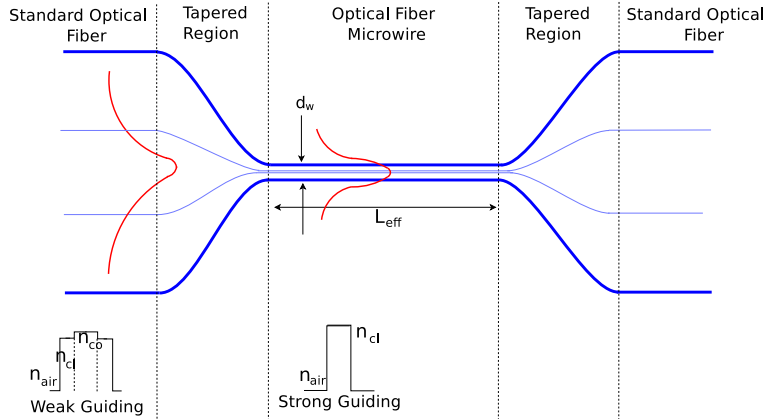


Figure 1. Schematic representation of an OFM.

The maximum efficiency of the FWM process is obtained for the maximum confinement of the optical fields, where it verifies an increase on the nonlinear coefficient ( $\gamma$ ). However, the high index contrast and the small diameter, can lead to the existence of several guiding modes. In this regime the strong guiding must be considered and the dispersion equation can be solved for the two layers vectorial model, considering one silica core and an infinite air cladding.

The model assumes that the field in the two layers can be described by:

$$E(r, \phi, z, t) = \sum_m E_m(r, \phi) \exp[i(\beta_m z - \omega t)] , \quad (1)$$

where  $E_m(r, \phi)$  and  $\beta_m$  are the transversal distribution of the electrical field and the propagation constant for the  $m$  order mode, respectively. The parameters  $r$ ,  $\phi$ ,  $z$  and  $t$  are the radial, azimuthal, longitudinal and temporal components, respectively, and  $\omega$  is the angular frequency.<sup>9</sup> The OFMs can support several modes, which can be hybrid (HE and EH), transversal electrical (TE), or transversal magnetic (TM).<sup>10</sup> The OFMs considered have

a refractive index equal to fiber cladding refractive index (silica), where cladding is an infinite layer of air. The guided modes in the OFM are found by numerically solving the dispersion equation.<sup>9</sup> The material dispersion is accounted by using the Sellmeier equation.<sup>11</sup>

## 2.1 FWM process

Analogously to single mode optical fibers, in single mode OFMs the SFWM process is also highly dependent on the phase-matching condition. This condition can be expanded in a Taylor series about the frequency  $\omega_k$ , at which the pulse spectrum is centered, obtaining,

$$\Delta\beta(\omega) = \Delta\beta_0 + \Delta\beta_1(\omega - \omega_k) + \frac{1}{2!}\Delta\beta_2(\omega - \omega_k)^2 + \frac{1}{3!}\Delta\beta_3(\omega - \omega_k)^3 + O(4), \quad (2)$$

where  $\beta_0 = kn_{eff}$ , with  $k$  and  $n_{eff}$  being the wave vector and the effective refractive index, respectively,  $\beta_1 = c/n_g$ , being  $c$  the speed of light and  $n_g$  the group index,  $\beta_2 = -\frac{2\pi c}{\omega_k^2}D(\omega_k)$ , with  $D$  representing the dispersion, and  $\beta_3 = \frac{(2\pi c)^2}{\omega_k^4}S(\omega_k) + \frac{4\pi c}{\omega_k^3}D(\omega_k)$ , with  $S$  being the differential dispersion.<sup>11</sup> From the 4th order, terms are too small and then can be neglected.

In Table 1, we present the values for  $\beta_4$  and  $\beta_5$  for several radii of an OFM, and for a dispersion-shifted fiber (DSF), for  $\lambda = 1550$  nm. It's observed from Table 1 that the order of the values of an OFM and a DSF is

Table 1. Comparison between the values of  $\beta_4$  and  $\beta_5$  for several radii of an OFM and DSF, for  $\lambda = 1550$  nm.

	Diameter ( $\mu\text{m}$ )	$\beta_4$ ( $\text{m}^{-5}$ )	$\beta_5$ ( $\text{m}^{-6}$ )
OFM	0.60	$-1.0317 \times 10^{-52}$	$2.3026 \times 10^{-67}$
	1.10	$7.3310 \times 10^{-53}$	$-1.6838 \times 10^{-67}$
	1.20	$5.6033 \times 10^{-53}$	$-3.2686 \times 10^{-67}$
	1.30	$3.9178 \times 10^{-53}$	$-3.0872 \times 10^{-67}$
	2.00	$-4.4193 \times 10^{-55}$	$-4.6645 \times 10^{-68}$
DSF <sup>12</sup>	-	$1.1786 \times 10^{-54}$	$2.6946 \times 10^{-68}$

relatively close, about one order of difference, which allows to expand the phase-matching condition in Taylor series. Comparing the values for the different diameters of the OFM, it can be seen that there is only a slightly variation between them, verifying that the expansion can be applied in any case.

Writing (2) in terms of  $\lambda$ , and solving it around  $\lambda_k$ , we get,

$$\Delta\beta = -\frac{2\pi c\lambda_k^2}{\lambda_p^3\lambda_s^2}(\lambda_p - \lambda_s)^2(\lambda_p D(\lambda_k) + \lambda_k(\lambda_p - \lambda_k))S(\lambda_k), \quad (3)$$

where  $\lambda_p$ ,  $\lambda_s$ , and  $\lambda_k$  are the pump, signal and central wavelengths, respectively.<sup>11,13</sup> In order to minimize the phase-matching condition, the dispersion must be zero.<sup>11</sup> Then, when the pump wavelength goes close to the ZDW ( $\lambda_0 = \lambda_k$ ), the phase-matching condition is verified, and the efficiency grows exponentially. However, for small deviations of  $\lambda_p$  in relation to  $\lambda_0$ , it verifies that the efficiency of the process decreases drastically.<sup>14,15</sup> For a ZDW value,  $\lambda_0 = 1550$  nm, the OFM target diameter was calculated as  $\Phi_t \approx 1.2315$   $\mu\text{m}$ , and the convergence radius of the series also, verifying that it is valid for a wavelength band of about 50 nm.

The efficiency of the FWM process can be written as,

$$P_i(L, \Delta\beta) = (\gamma P_p(0)L_{eff})^2 P_s(0) \exp(-\alpha L) \left[ \frac{\alpha^2}{\alpha^2 + \Delta\beta^2} \left( 1 + \frac{4 \exp(-\alpha L) \sin\left(\frac{\Delta\beta L}{2}\right)^2}{(1 - \exp(-\alpha L))^2} \right) \right], \quad (4)$$

where  $P_p$ ,  $P_s$ ,  $\gamma$ ,  $\alpha$  and  $L$  are the pump and signal powers, the nonlinear coefficient, the attenuation and the fiber length, respectively. Due to the low attenuation of silica and the reduced lengths of the OFM, the effective length,  $L_{eff}$ , can be approached to the OFM length. However, for tapered optical fibers this approximation may be not valid if the tapered regions have a considerable optical loss. From (4) we can see that the efficiency of the FWM process is highly dependent on the phase matching condition.

## 2.2 Mode analysis

In this paper we focus the analysis around the telecom wavelengths. The OFMs with the ZDW for the fundamental mode around 1550 nm wavelength band, have its diameter at  $\Phi \approx 1.2 \mu\text{m}$ .<sup>6</sup> For this diameter, the OFMs present only three possible modes ( $\text{HE}_{11}$ ,  $\text{HE}_{21}$  and  $\text{TM}_{01}$ ), once for the other ones the cut-off frequency have been achieved, as can be seen from Fig. 2 a).

In Fig. 2 b) we show the dispersion as a function of the wavelength, for the first three propagation modes presented in Fig. 2 a). As can be seen from Fig. 2 b), only the fundamental mode ( $\text{HE}_{11}$ ) has a ZDW close to 1550 nm. Therefore, only the photons that propagate in this mode are the ones that will respect the phase-matching condition in the FWM process. In this way, for such diameter, only the fundamental mode ( $\text{HE}_{11}$ ) contributes for the FWM process. In Fig. 2 b), we can see that the ZDW of the fundamental mode is about 30% away from the ZDW of the closest mode. According to this, we have sure that only the ZDW of the fundamental mode is measured in the 1550 nm wavelength band, if the scanning is performed in the decreasing way of  $\lambda$ .

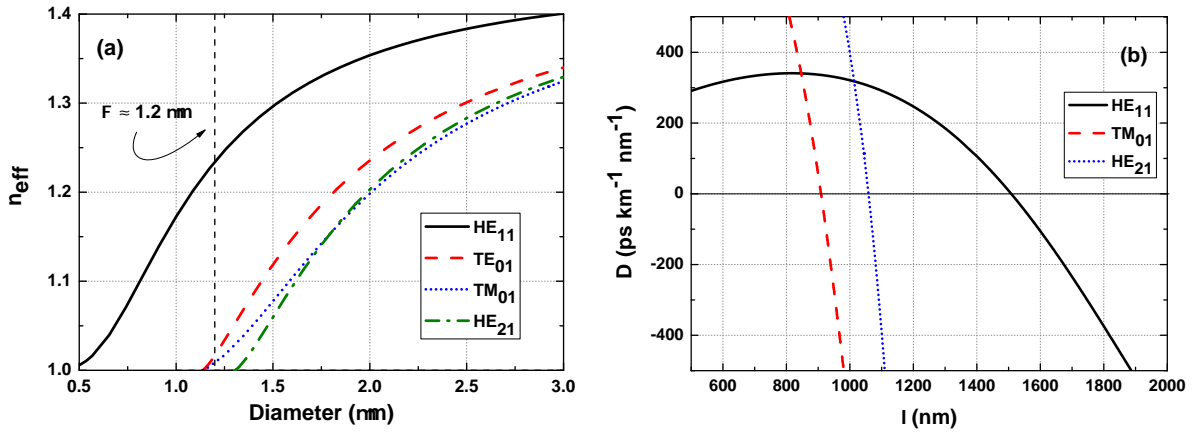


Figure 2. a) Effective refractive index as a function of the OFM diameter, for four different propagation modes. As can be seen for a diameter,  $\Phi = 1.2 \mu\text{m}$ , only three propagating modes ( $\text{HE}_{11}$ ,  $\text{TE}_{01}$  and  $\text{TM}_{01}$ ) are allowed; b) Dispersion as a function of the wavelength for the only three modes that remain for a diameter  $\Phi = 1.2 \mu\text{m}$ . It can be seen that only the fundamental mode ( $\text{HE}_{11}$ ) has a ZDW at the 1550 nm wavelength band.

## 3. MEASUREMENT TECHNIQUE

The optical field confinement in a waveguide with a diameter of the same order of the optical field's wavelength, leads to an important part of the wave spread in the outer guide. Therefore, the waveguide dispersion becomes dominant, which allows that the second ZDW occurs near the region of maximum confinement. Regardless of the spectral region where the ZDW occurs, the optical field is strongly confined in the waveguide, so that the nonlinear coefficient is high, which leads to a more efficient FWM process, even for very small interaction lengths.

The FWM process can be obtained using two input signals, and the generated signal, the *idler*, must verify the relation  $\lambda_i = (\lambda_p \lambda_s) / (\lambda_s + \lambda_i)$ .<sup>11</sup> Hence, the idler gain will be as bigger as close the pump is launched to the ZDW. In Fig. 4 a), we show the schematic of the FWM process using two input signals. The wavelength separation between pump and signal ( $\Delta\lambda_{p-s}$ ), and between pump and idler ( $\Delta\lambda_{p-i}$ ) is constant. The efficiency of the FWM process will be as higher as close to the ZDW the pump is launched, thus it allows to estimate the ZDW. In Fig. 3 a) it is shown the dispersion as a function of the wavelength for OFMs with different diameters, for the fundamental mode. From Fig. 3 a), we can see that as bigger the diameter, smaller the curvature of the function that describes the dispersion of the OFM, verifying that for diameters greater than  $1.3 \mu\text{m}$ , the second ZWD is very far from the common wavelengths used in telecommunications. For diameters smaller than  $1.2 \mu\text{m}$ , the value for the ZDW decreases, approaching the visible wavelengths.

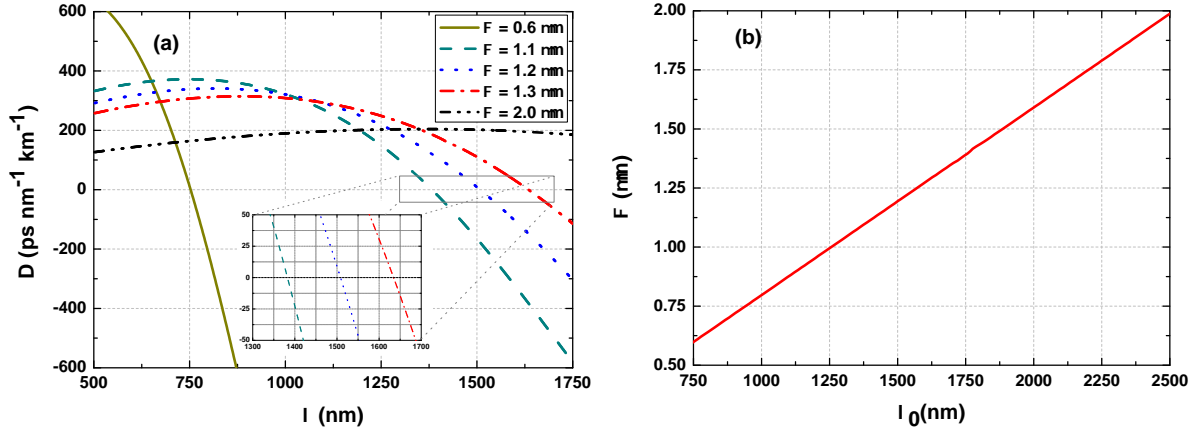


Figure 3. a) Dispersion as a function of the wavelength for different values of OFM diameters, for the fundamental mode. In the detailed picture it can be seen that when  $\Phi \approx 1.2 \mu\text{m}$ , dispersion will be zero for  $\lambda \approx 1550 \text{ nm}$ ; b) Variation of the ZDW with the OFM diameter.

In Fig. 3 we can see the high variation of the ZDW with the diameter. In this way, the empirical determination of the ZDW can be used to obtain the OFM diameter with high accuracy, analogously to other methods already presented.<sup>9</sup> This method has the advantages of being nondestructive, to present high resolution and to be relatively easy to implement. In Fig. 3 b), we show the variation of the OFM diameter with the ZDW. The linear behaviour presented can provide an accurate relation between these two parameters.

### 3.1 Experimental implementation

In Fig. 4 we show the schematic of how the ZDW can be measured. We change the pump wavelength, keeping the separation between the pump and signal constant. Then, we measured the idler power a function of the pump wavelength. The theoretical curve that describes the FWM process, given by (4), can be fitted to experimental data, thus enabling us to estimate the ZDW.

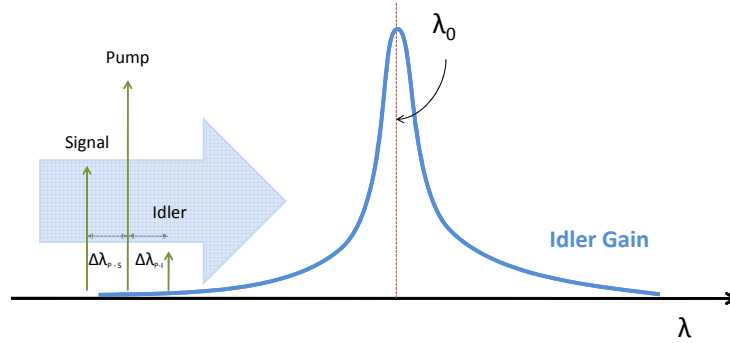


Figure 4. Schematics of the FWM process using two input signals. The wavelength separation between pump and signals is constant. As the pump goes to close the ZDW, the gain increase.

In this way, we propose the experimental setup represented in Fig. 5. A pump, from a continuous light-wave (CW) tunable laser source (TLS) is mixed with a 3 dB coupler with another optical field from an external cavity laser (ECL). The two optical fields pass through a linear polarizer (LP), to assure that they are sent to the OFM in the same linear polarization state. At the OFM output, the power of the three optical fields are analyzed in a optical spectrum analyser (OSA).

Preliminary experimental tests of the proposed technique show that its needed a thin scanning around the ZDW. The theoretical description of the FWM process shows that this is extremely sensitive to the ZDW, so the

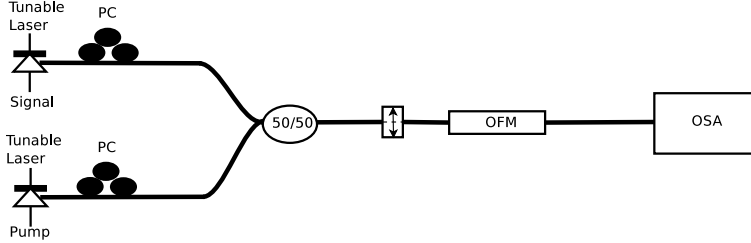


Figure 5. Experimental setup proposed to FWM measurement.

scanning should be done by moving the pump and the signal at most a few nanometers (ideally, one nanometer). Despite the high  $\gamma$ , the small length of the OFMs restricts the use of this technique. In this way, we need relatively high powers for the pump and the signal. This method requires stable lasers and large wavelength bands. However, the major limitation of this technique lies in the efficiency of FWM process.

The length, the nonlinear coefficient of the OFM and the pump and signal input powers, are crucial parameters to define the applicability of the technique presented. In this way, we propose the  $\gamma LP$ , which allows to estimate the applicability of the technique presented. For  $\gamma LP \geq 2.5 \times 10^{-2}$  is possible to measure the idler, and consequently it can be estimated the ZDW, the nonlinear coefficient and the slope dispersion for the ZDW. This value is estimated taking into account the sensitivity of the measurement devices (OSA, Agilent-8384). Usually, the length of the OFMs is relatively small. However, various experimental systems capable of produce OFMs with several meters were already presented.<sup>16</sup>

The nonlinear parameter can be incremented in several orders for OFMs built from other materials, with the nonlinear refractive index higher. However, is still possible to use an high pump power, that increase the idler gain. This can be easily used to identify the ZDW. Numerical results show that the pump-signal wavelength separation is also very important, in the efficiency of FWM process. The results advise the use of a wavelength separation greater than 10 nm.

#### 4. CONCLUSIONS

A nondestructive technique for measuring the diameter of OFMs, using the SFWM process is proposed, and a theoretical study of the ZDW as a function of the OFMs diameter is presented. The efficiency of the SFWM as function of the separation between the ZDW and the pump wavelength is used to estimate the diameter of the OFMs.

A numerical analysis is performed, and the ideal parameters for easy experimental measurements are presented and discussed. The  $\gamma PL$  criterion is proposed to define the applicability of the technique, and a minimum value is established (taking into account the sensitivity of the measurement instruments available in our laboratory).

This method can be considered of high precision, since the efficiency of the FWM process is sufficiently high, allowing to identify clearly the idler gain. For a resolution in determining the ZDW of 3 nm we can achieve a resolution in determining the diameter of 4.4 nm. This high accuracy is only possible using SEM (Scanning Electron Microscope) or through the third harmonic generation.<sup>9</sup>

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