Experimental Characterization of the Photon Statistics of Four-Wave Mixing Photon Source

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***Abstract*—We discuss the statistics of Poisson and thermal sources, and verify that as an approximation to a single-photon source, both Poissonian and thermal sources can be used, when the average number of photons per pulse is of the order of 0.1. An experimental characterization of the photon statistics of a single-photon source obtained from the four-wave mixing (FWM) process in optical fibers is presented. We employ an on/off avalanche photodetector operating in the Geiger mode, and use the maximum-likelihood estimation (MLE) method and the expectation-maximization (EM) algorithm to reconstruct the data. We find that the statistics of the source goes from thermal, at a low signal power regime, to Poissonian, in a high signal power regime.**

***Index Terms*—Photon Statistics, Optical Mixing, Maximum-Likelihood Estimation, Avalanche Photodetector, Optical Communication.**

I. INTRODUCTION

Quantum key distribution (QKD) is one of the most successful achievements of quantum mechanical theory [1]. Several single-photon sources and detectors have been developed, and QKD has evolved from point-to-point fiber links, to optical fiber networks [1–9]. As one of the most simple to implement, an attenuated laser source has been used as an approximation to a single-photon source [5, 10–12]. Recently, the stimulated four-wave mixing (FWM) process in optical fibers has been also used to obtain an approximation to a single-photon source [13–21].

The characterization of the photon statistics of a single-photon source is of major importance, since it provides fundamental information on its nature [1]. The photon statistics characterization has been performed for several photon sources, and using different techniques. A method based on quantum tomography, for measuring the photon statistics of twin beams emerging from a nondegenerate optical parametric oscillator, was presented in [22]. Using the same method, the statistical characterization of the amplified signal at the fiber-optic parametric amplifier (FOPA) output was presented in [23]. In [24], photon detectors were used as photon counters, in order to determine the statistics of a single-photon source obtained from a coherent state. Photomultiplier tubes were also employed to characterize the statistics of quantum states in the continuous-variable regime [25]. A different approach to estimate the photon statistics of an optical source was recently presented [26]. In that work, on/off avalanche photodetectors operating in the Geiger mode, assisted by the maximum-likelihood estimation (MLE) method and the expectation-maximization (EM) algorithm were used.

A theoretical characterization of the statistics of a single-photon source based on the FWM process in optical fibers was recently presented [27, 28]. In this work we present an experimental characterization of the photon statistics of a single-photon source based on the FWM process in optical fibers. We employ an on/off avalanche photodetector assisted by the MLE method and the EM algorithm to numerically reconstruct the photon statistics. A characterization of Poissonian and thermal sources in terms of the probability distribution of the number of photons is also presented and discussed.

This paper is organized in five sections. In section II, we describe the experimental measurement method and the numerical reconstruction method used to obtain the photon statistics of the FWM process in optical fibers. In section III, we discuss the statistical distributions of the Poissonian and the thermal sources, and compare them in terms of fluctuations in several regimes. In section IV, we present and discuss the results on the reconstruction of the photon statistics of the FWM process. Finally, in section V we show our conclusions.

II. MEASUREMENT METHOD

A. Description of the Experimental Measurement Method

In Fig. 1, we present the schematics of the experimental setup used to determine the statistics of the photons that are generated from the proces of FWM.

A pump at $\lambda_{\mathrm{p}} = 1550.92$ nm, from a continuous wave (CW) tunable laser source (TLS), was multiplexed (MUX) with a signal from an external cavity laser (ECL) centered at $\lambda_{\mathrm{s}} = 1547.72$ nm. The signal field was modulated into pulses with a full-width at half maximum (FWHM) of approximately 1 ns and a repetition rate of 1.22 MHz, using a Mach-Zehnder modulator (Mod). The polarization controllers (PCs) and the linear polarizer (LP) were used to send both fields co-polarized to the dispersion-shifted fiber (DSF), where an idler wave was generated at $\lambda_{\mathrm{i}} = 1554.13$ nm, which follows the relation [29],

$$\lambda_{\mathrm{i}} = \frac{\lambda_{\mathrm{p}}\lambda_{\mathrm{s}}}{2\lambda_{\mathrm{s}} - \lambda_{\mathrm{p}}}\,. \tag{1}$$

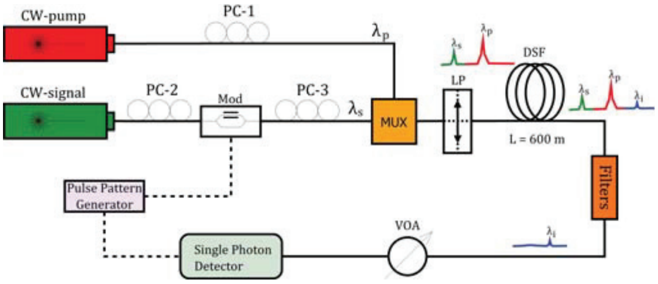

Fig. 1. Schematics of the experimental setup. The dashed lines represent electrical signals and the solid lines the optical path.

The DSF is characterized by a nonlinear parameter $\gamma = 2.3$ W^{-1}km^{-1}, a zero-dispersion wavelength $\lambda_0 \approx 1550$ nm, a dispersion slope at λ_0 of $dD_c/d\lambda = 0.071$ ps/(nm^2-km), and a length $L = 600$ m. At the output of the fiber, the pump and signal photons were suppressed by two cascaded flat-top dense wavelength-division multiplexing (DWDM) optical filters with a FWHM of 100 GHz, which provide a total isolation of about 100 dB. The idler photons passed through the filters and reached a variable optical attenuator (VOA) and a single photon detector module (SPDM), model id201 from ID Quantique, operating in the Geiger mode [30, 31]. In the SPDM, a gate duration of 5 ns was employed, and the deadtime was set to 10 μs, in order to avoid afterpulses. The quantum detection efficiency of the SPDM was measured as $\eta_D = 7.1\%$, and the probability of having dark counts as $P_{dc} = 2.55\times10^{-5}$. The outcome of the SPDM was the number of clicks in a time interval of 20 s, and the total number of gates that were open by the detector in the same time interval, for a chosen efficiency value on the VOA.

B. Numerical Reconstruction Method

The numerical method used for reconstructing the photon statistics through on/off detection (click/no-click events) was introduced in [32], and implemented in [26]. The statistics of the no-click events from the SPDM is given by,

$$p_\nu^{\text{off}}(\eta_\nu) = (1 - P_{\text{dc}}) \sum_{n=0}^{N} (1-\eta_\nu)^n \rho_n \,, \tag{2}$$

where η_ν, with $\nu = 1 \ldots K$, are the values of the combined efficiencies of the SPDM and the VOA, and ρ_n is the probability to obtain n photons. Since the model is linear and the parameters are positive, the solution for ρ_n can be obtained by the MLE method, through the EM algorithm,

$$\rho_n^{(i+1)} = \frac{\rho_n^{(i)}}{\sum_{j=1}^{K} A_{jn}} \sum_{\nu=1}^{K} f_\nu \frac{A_{\nu n}}{p_\nu^{\text{off}}[\{\rho_n^{(i)}\}]} \,, \tag{3}$$

where $A_{\nu n} = (1 - P_{dc})(1-\eta_\nu)^n$, f_ν denotes the experimental frequencies of the no-click events for the efficiency η_ν, and $p_\nu^{\text{off}}[\{\rho_n^{(i)}\}]$ is the no-click probability obtained from the reconstructed distribution $\{\rho_n^{(i)}\}$. In our algorithm, we set the upper limit in the sum (2) to $N = K - 1 = 30$. This allows the verification of the condition,

$$\sum_{n=0}^{N} \rho_n \approx 1 \,. \tag{4}$$

The reconstructions were performed using $N_i = 1.22\times10^6$, *i.e.*, making the number of iterations equal to the pulse rate [26]. As a measure of accuracy between the theoretical and the reconstructed distributions we adopted the fidelity [26],

$$G \;=\; \sum_{n=0}^{N} \sqrt{\rho_n \rho_n^{(N_i)}} \,. \tag{5}$$

The reliability of this method was confirmed in [33–35]. In those works, authors show that this numerical method is robust to fluctuations on the values of η.

III. PROBABILITY DISTRIBUTION OF THERMAL AND POISSONIAN LIGHT SOURCES

In order to know the true nature of a photon source, the statistical properties of the light of the source must be determined. In that sense, it is important to compare the light emitted by a black-body or a lamp, with the light from a stable laser. For the first case, known as thermal light, many independent emitters contribute for the final signal, that is a superposition of many incoherent waves. The statistics of the light generated with these sources is known as thermal [36, 37]. The thermal statistics is described as [38–40],

$$P_{\text{th}}(n, \mu) = \frac{\mu^n}{(1+\mu)^{n+1}} \,, \tag{6}$$

where $P_{\text{th}}(n, \mu)$ is the probability of measuring n photons, given a mean measured photon number μ. The light from a stable laser is a wave with constant amplitude and phase, and is represented as a coherent state [36, 37]. The number of photons in a coherent state is a variable that fluctuates according to a Poissonian distribution, which is given by [41],

$$P_{\text{P}}(n, \mu) = \frac{\mu^n}{n!} e^{-\mu} \,. \tag{7}$$

The photon statistics can be characterized in terms of the second-order coherence function. That function is defined as [42],

$$g^{(2)}(0) = 1 + \frac{Q}{\mu} \,, \tag{8}$$

where the Mandel Q parameter is written as,

$$Q = \frac{\text{Var}}{\mu} - 1 \,. \tag{9}$$

The variance and the mean photon number are obtained from,

$$\text{Var}^{(N_i)} = \sum_{n=0}^{N} \left(n - \mu^{(N_i)}\right)^2 \rho_n^{(N_i)} \,, \tag{10}$$

and,

$$\mu^{(N_i)} = \sum_{n=0}^{N} n \rho_n^{(N_i)} \,. \tag{11}$$

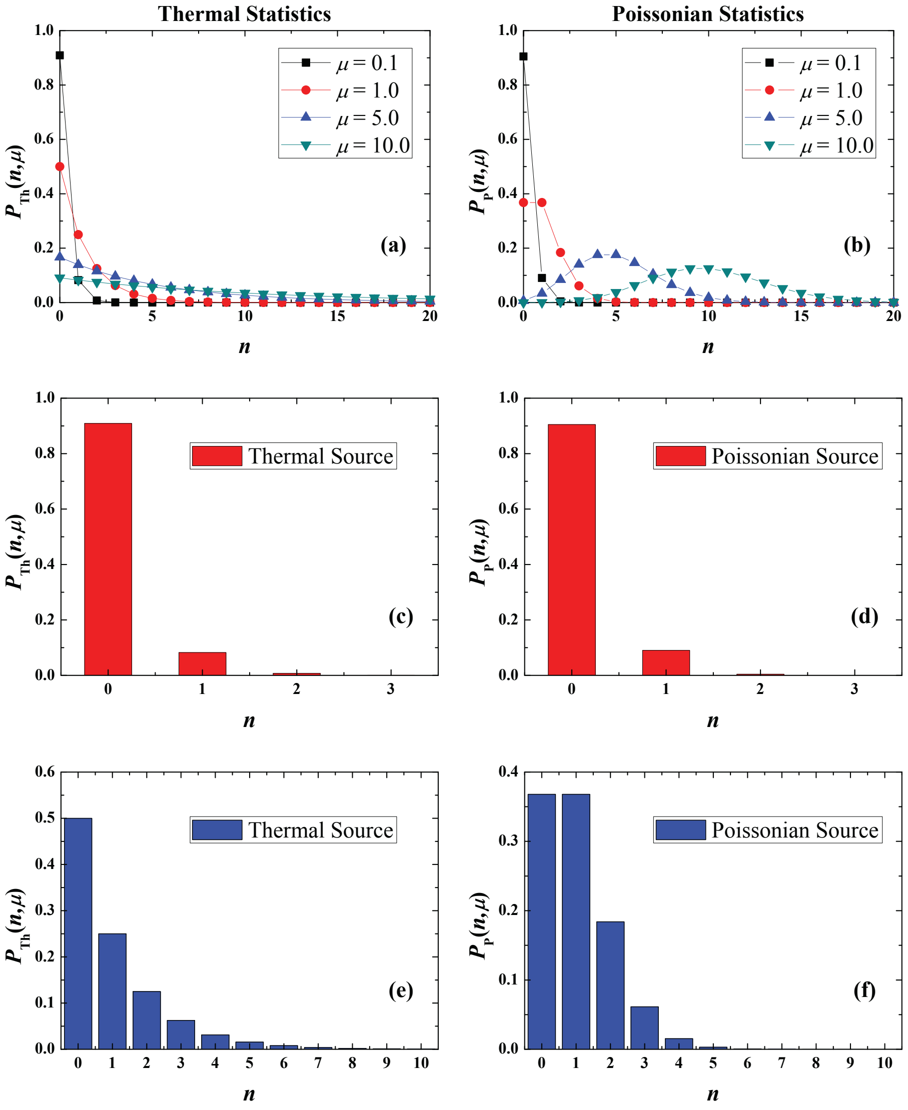

Fig. 2. Probability distribution of the number of photons for two light sources. In 2(a)-2(b) are presented the results for several average numbers of photons per pulse μ, in 2(c)-2(d) $\mu = 0.1$, and in 2(e)-2(f) $\mu = 1$. The thermal source presents a large number of fluctuations in relation to the mean photon number, due to the Bose-Einstein statistics of black-body radiation. The Poissonian or coherent source is narrower than the thermal one, but still with a strong number of fluctuations, called photon noise.

A neutral $g^{(2)}(0)$ corresponds to a perfect single-photon source, and $g^{(2)}(0) < 1$ determines the nonclassical nature of the optical field. Otherwise, $g^{(2)}(0) \geq 1$ represents the classical nature of the field being measured. In addition, $g^{(2)}(0) = 1$ corresponds to a Poissonian statistics, and $g^{(2)}(0) = 2$ determines a source with thermal statistics, such as the spontaneous Raman scattering [42–44].

In Figs. 2(a)-2(b), we plot the probability distribution for photon sources that follow thermal and Poissonian statistics, for different μ. We can see that in the thermal statistics, the state with zero photons ($n = 0$) always has the largest probability of occupation. Therefore, for a sufficiently high number of photons, the statistics are barely indistinguishable. In the Poissonian statistics, the number of photons obeys to the central limit theorem for sufficiently large numbers [36].

In Figs. 2(c)-2(d), we compare the probability distribution of the number of photons for photon sources following thermal and Poissonian statistics, when $\mu = 0.1$, and in Figs. 2(e)-2(f) when $\mu = 1$. In Figs. 2(c)-2(d) we can see that, the probability distribution is very similar for both photon sources. Therefore, both can be used as approximations to single-photon sources, when $\mu \approx 0.1$. In Table I we present the probability distributions of the number of photons for the two sources, when $\mu = 0.1$. From Table I, we can see that both sources present equivalent probability distributions for $n = 0$ and $n = 1$, and also that the probability of those sources to produce more than one photon per pulse when $\mu = 0.1$, is very small.

TABLE I

PROBABILITY DISTRIBUTIONS OF THE NUMBER OF PHOTONS FOR THERMAL AND POISSONIAN SOURCES, WHEN $\mu = 0.1$.

n	$P_{\text{th}}(n, 0.1)$	$P_{\text{P}}(n, 0.1)$
0	0.9091	0.9048
1	0.0826	0.0905
2	$7.5\ \times 10^{-3}$	$4.52\ \times 10^{-3}$
3	$6.83\ \times 10^{-4}$	$1.51\ \times 10^{-4}$

According to Fig. 2(e), where $\mu = 1$, we can see that the state with zero photons always present the largest probability of occupation, and the thermal distribution follows the relation, $1^n/2^{n+1}$. This distribution is therefore very far from a true single-photon source, which should present a sharp maximum for $n = 1$. The fluctuations may be seen as arising from the boson character of the photons [36]. In Fig. 2(f), we can see that the probability of the state with zero photons is equal to the probability of the state to have one photon.

In Table II are presented the probability distributions of the number of photons for the two sources, when $\mu = 1$.

TABLE II

PROBABILITY DISTRIBUTIONS OF THE NUMBER OF PHOTONS FOR THERMAL AND POISSONIAN SOURCES, WHEN $\mu = 1$.

n	$P_{\text{th}}(n, 1)$	$P_{\text{P}}(n, 1)$
0	0.5	0.36787944
1	0.25	0.36787944
2	0.125	0.18393972
3	0.0625	0.06131324
4	0.03125	0.01532831
5	0.015625	0.00306566
6	0.0078125	$5.1094367\ \times 10^{-4}$
7	0.00390625	$7.2991953\ \times 10^{-5}$
8	0.00195313	$9.1239941\ \times 10^{-6}$
9	$9.765625\ \times 10^{-4}$	$1.0137771\ \times 10^{-6}$
10	$4.8828125\ \times 10^{-4}$	$1.0137771\ \times 10^{-7}$

From Table II we can see more clearly the effect of fluctuations, since when $n = 10$, for example, the probability of having thermal photons is 3 orders higher than Poissonian photons.

IV. EXPERIMENTAL CHARACTERIZATION OF FOUR-WAVE MIXING PHOTON SOURCE STATISTICS

In this section, we present the numerical reconstruction of the statistics of the photons generated from a photon source obtained from the FWM process inside an optical fiber.

The setup used in this experiment is presented in Fig. 1. We have measured the single-photon counts in the SPDM as a function of the VOA efficiency, and then we have used the MLE method and the EM algorithm to reconstruct the photon statistics.

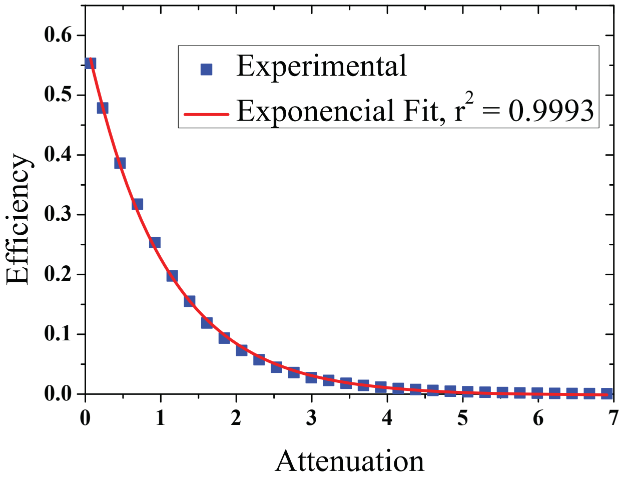

Fig. 3. Efficiency of the VOA as a function of its attenuation.

In Fig. 3 we present the efficiency of the VOA as a function of its attenuation, for the 31 points that we have measured, and that were used in the numerical reconstruction method.

An exponential decay behavior can be observed, where the curve follows the equation, $\eta = 0.602 \times \exp\left(-\alpha/1.032\right) - 0.002$, with α being the attenuation of the VOA.

The numerical reconstruction was obtained for seven different signal powers, considering the pump power fixed. In Fig. 4(a), we plot the fidelity values obtained from each numerical reconstruction as a function of the average number of photons per pulse at the output of the source, for the two distributions tested. As can be seen from Fig. 4(a), when the pulse carries a low number of photons per pulse, *i.e.* $\mu \lesssim 2$, the source follows a thermal statistics. As μ is increased, the Poissonian statistics arises. This can be seen more clearly from the second-order coherence function, that is plotted in Fig. 4(b). From the results obtained, we can see that the statistics of the source goes from thermal, at a low power regime (which corresponds also to a low μ), to Poissonian, in a high power regime (higher μ). A thermal statistics can be explained by the fact that spontaneous processes dominate in a low power regime. When μ is increased in the signal, the stimulated process becomes more significant, and the idler photons follow a Poissonian statistics.

The pump and signal powers at the input of the fiber and the average number of photons per pulse at the fiber output, on the idler wave, obtained from the numerical reconstructions, are presented in Table III.

V. CONCLUSIONS

We have discussed the statistical distributions of the Poissonian and the thermal photon sources, and compared them. When using $\mu \approx 0.1$, both sources can be used as approximations to single-photon sources. In the thermal statistics the empty state always has the largest probability of occupation, and in the Poissonian statistics the probability of having an empty state is equal to the probability of having a state

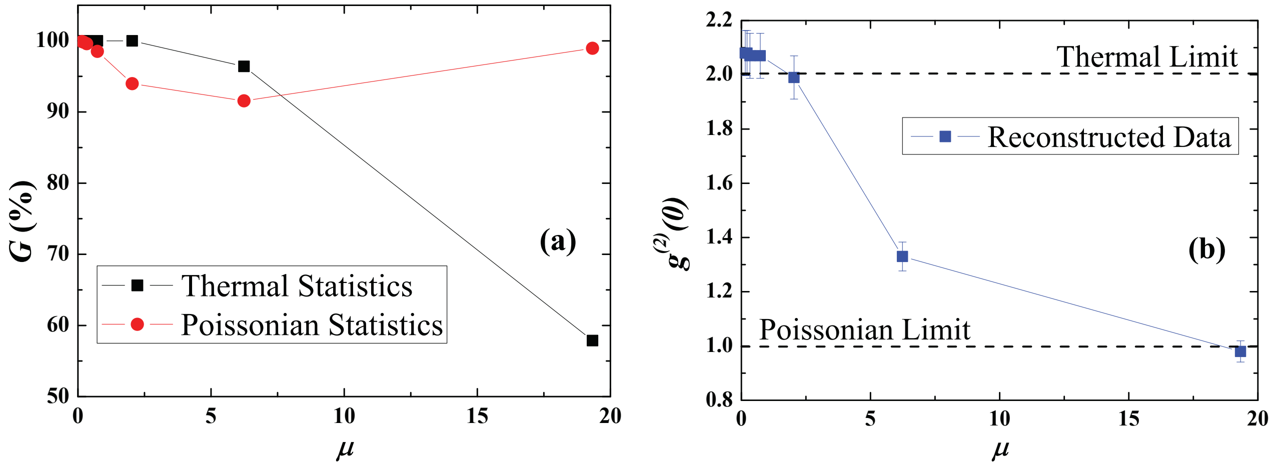

Fig. 4. (a) Fidelity and (b) second-order coherence function obtained from the reconstructed data as a function of the average number of photons per pulse at the output of the source. The pump and signal powers used in each case are presented in Table III. The error bars in Fig. 4(b) represent the fluctuations on the experimental data.

TABLE III

PUMP AND SIGNAL POWERS AT THE INPUT OF THE FIBER, AND CORRESPONDENT μ AT THE OUTPUT OF THE SOURCE. THE FIDELITIES OBTAINED FROM THERMAL, G_{TH}, AND POISSONIAN, G_{P}, RECONSTRUCTIONS, ALONG WITH THE SECOND-ORDER COHERENCE FUNCTION, $g^{(2)}(0)$, ARE ALSO PRESENTED.

$P_{\mathrm{p}}(0) = 4.2$ [dBm]				
$P_{\mathrm{s}}(0)$ [dBm]	μ	G_{th} [%]	G_{P} [%]	$g^{(2)}(0)$
-37.2	0.15	99.99	99.90	2.08
-39.2	0.22	99.99	99.81	2.08
-33.2	0.33	99.99	99.59	2.07
-28.9	0.73	99.99	98.52	2.07
-24.2	2.04	100	93.97	1.99
-39.2	6.24	96.41	91.57	1.33
-18.9	19.33	57.87	98.95	0.98

with only one photon. Also, the probability of having pulses carrying two or more photons is not negligible, which may compromise the security in QKD systems.

We also presented an experimental characterization of the photon statistics of a single-photon source obtained through the FWM process in optical fibers. We have measured the single-photon counts in the SPDM as a function of the VOA efficiency, and then we have used the MLE method and the EM algorithm to reconstruct the statistics of the photons. The numerical reconstruction was performed for seven different signal powers at the input of the DSF, considering the pump power fixed. We found that the statistics of the source goes from thermal at a low power regime, corresponding to a low number of photons per pulse ($\mu \lesssim 2$), to Poissonian, in a high power regime, corresponding to an higher number of photons per pulse ($\mu \gtrsim 2$).

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